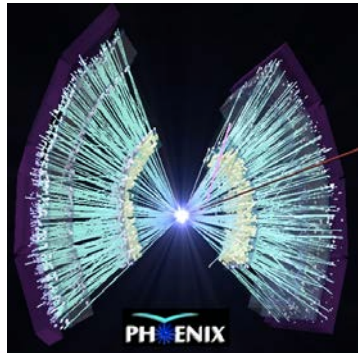


# Issues of Constituent Quark generation with Monte Carlo Glauber Models in p+p and Au+Au measurements

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16<sup>th</sup> Zimanyi School  
Budapest Hungary  
December 5-9 2016



PHENIX2014  $E_T$  distributions  
from PRC89(2014)044905  
 $dE_T/d\eta$   $dN_{ch}/d\eta$  from  
PRC93(2016)024901



# Extreme Independent Models

- **Extreme-Independent models:** separate nuclear geometry and fundamental elements of particle production.
- Nuclear Geometry represented by relative probability  $w_n$  per B+A interaction for a given number  $n$  of fundamental elements.
- I will discuss models with 3 different fundamental elements:
  - ✓ **Wounded Nucleon Model (WNM)** - number of participants  $N_{part}$
  - ✓ **Quark Part. Model (NQP)**, -number of constituent-quark participants  $N_{qp}$
  - ✓ **Additive Quark Model (AQM)**, color-strings between quark participants in projectile & target: constraint: one string per qp → **projectile quark participants**.
- AQM & NQP cannot be distinguished for symmetric collisions, since projectile and target have the same number of struck quarks. Need asymmetric collisions, *e.g.*, d+Au,

# Implementation

- The dynamics of the fundamental elementary process are taken from the data: e.g. the measured  $E_T$  distribution for a p-p collision represents: 2 participants (WNM); **a predictable convolution of constituent-quark-participants (Nqp)**; or projectile quark participants (AQM).
- The above bullet is why I like these models: a Glauber calculation and a p-p measurement provide a prediction for A+A **in the same detector!**

# Why constituent-quarks now?

# What are Constituent Quarks?

Constituent quarks are Gell-Mann's quarks from Phys. Lett. 8 (1964)214, proton=uud. These are relevant for static properties and soft physics, low  $Q^2 < 2 \text{ GeV}^2$ ; resolution  $> 0.14 \text{ fm}$

For hard-scattering,  $p_T > 2 \text{ GeV}/c$ ,  $Q^2 = 2p_T^2 > 8 \text{ GeV}^2$ , the partons ( $\sim$ massless current quarks, gluons and sea quarks) become visible

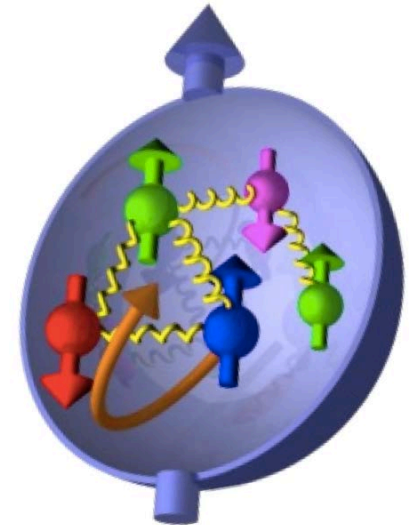


1.6fm

Resolution  $\sim 0.5 \text{ fm}$



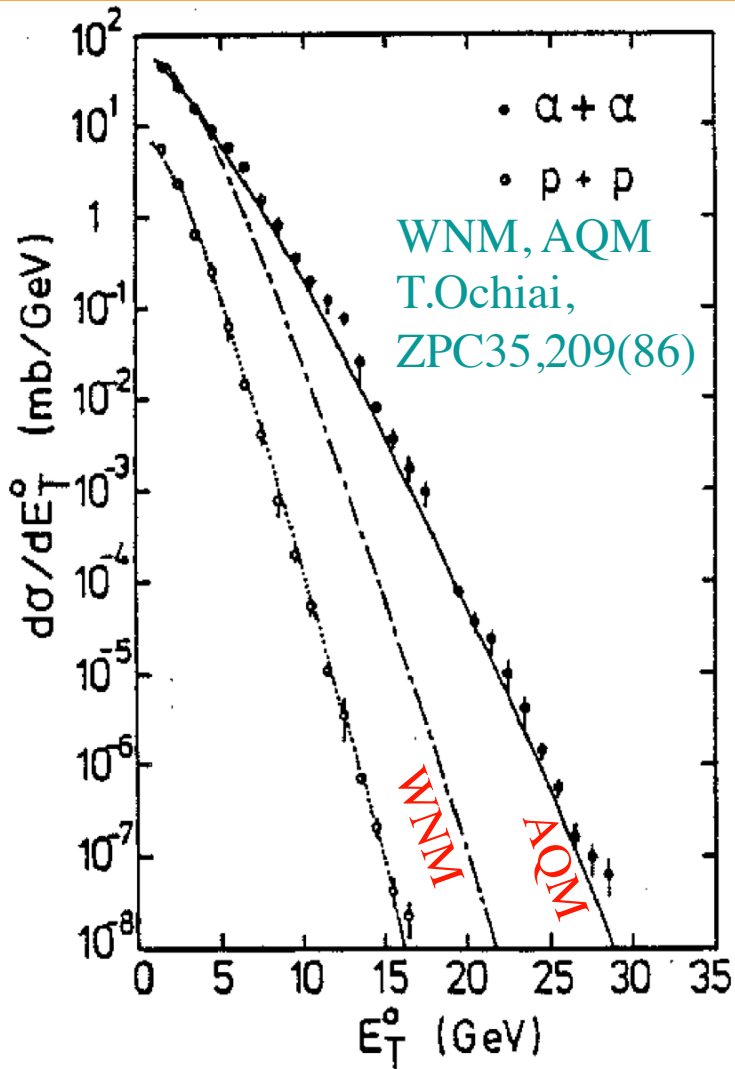
Resolution  $\sim 0.1 \text{ fm}$



Resolution  $< 0.07 \text{ fm}$

# From My First Quark Matter Talk 1984

## ISR-BCMOR- $\alpha\alpha$ $\sqrt{s_{NN}}=31\text{GeV}$ : WNM FAILS! AQM works



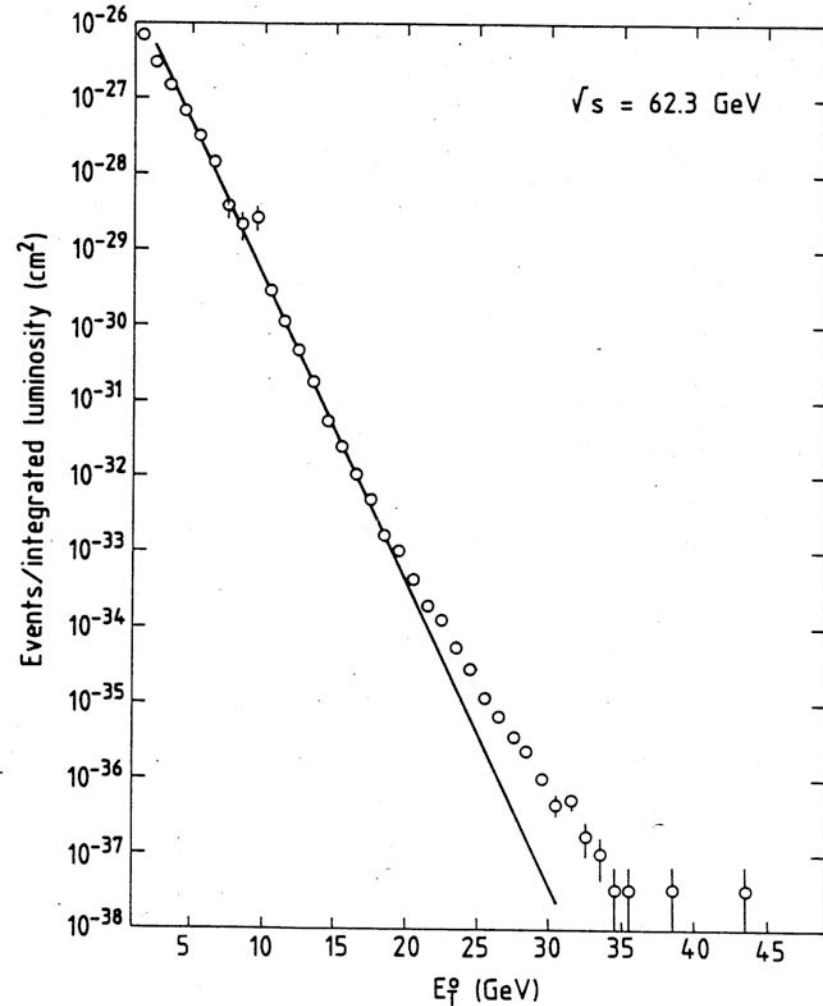
BCMOR PLB168(1986)158

WNM agrees with  $\alpha\alpha$  data for 1 order of magnitude but disagrees for the other 10 orders of magnitude.

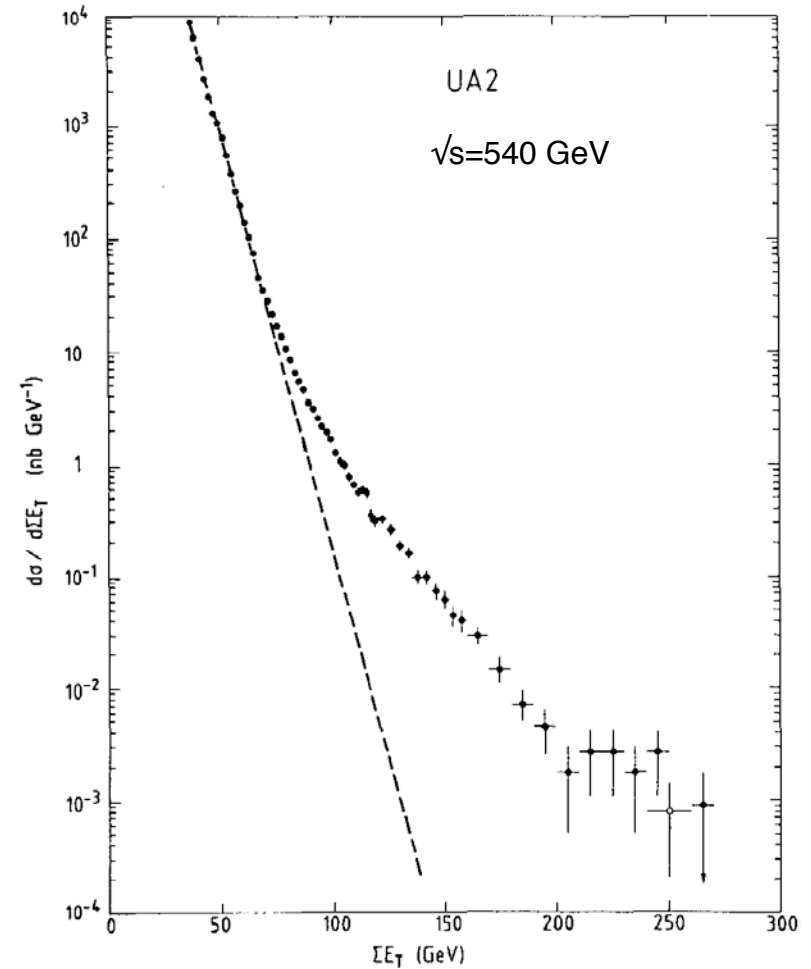
AQM ( $N_{qp}$ ) is in excellent agreement over the entire distribution.

A youngster, Bill Zajc, and other Penn collaborators claimed that failure of WNM was due to jets. BUT, in pp collisions  $E_T^0$  is dominated by soft physics, jet effects are not visible until four orders of magnitude down in cross section. For  $\alpha$ - $\alpha$  no jet effect in whole measured region [see CMOR Nucl.Phys B244(1984)1]

# Jets are a $<10^{-3}$ effect in p-p $E_T$ distributions

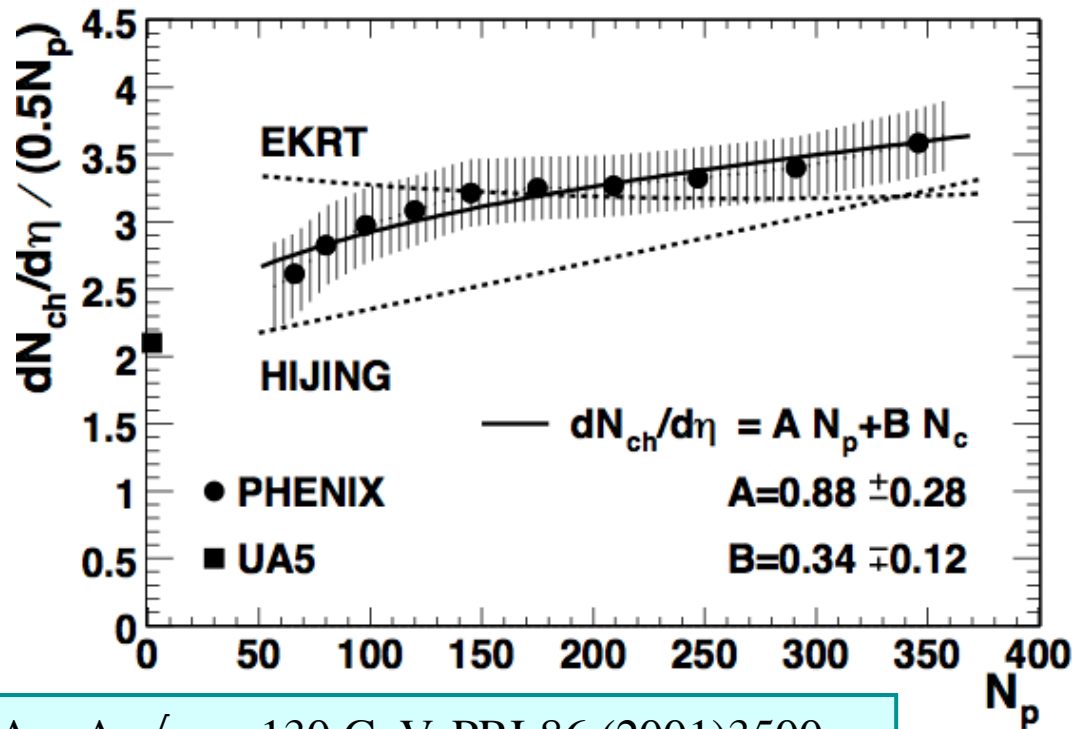


COR PLB**126**(1983)132  $E_T$  in  $\Delta\Phi=2\pi$ ,  $|\eta|<0.8$  EMCal. Break above 20 GeV is due to jets. Also see NuclPhys B**244**(1984)1



UA2 PLB**138**(1984)430 (from DiLella)  
Break from jets  $\sim 5$ -6 orders of magnitude down for  $E_T$  in  $\Delta\Phi=2\pi$ ,  $|\eta|<1.0$

# First PHENIX paper from RHIC evolution of mid-rapidity $dN_{ch}/d\eta$ with centrality, $N_{part}$



PHENIX Au+Au  $\sqrt{s_{NN}}=130$  GeV, PRL86 (2001)3500

Inspired by article in same issue [PRL86, 3496], PHENIX included the following fit:

$$dE_T^{AA}/d\eta = [(1 - x) \langle N_{part} \rangle dE_T^{pp}/d\eta/2 + x \langle N_{coll} \rangle dE_T^{pp}/d\eta]$$

The  $N_{coll}$  term implied a hard-scattering component for  $E_T$ , known to be absent in p-p

# Second PHENIX paper from RHIC evolution of mid-rapidity $dE_T/d\eta$ with centrality, $N_{part}$

PHENIX  $\sqrt{s_{NN}}=130$  GeV,  
PRL87 (2001)052301

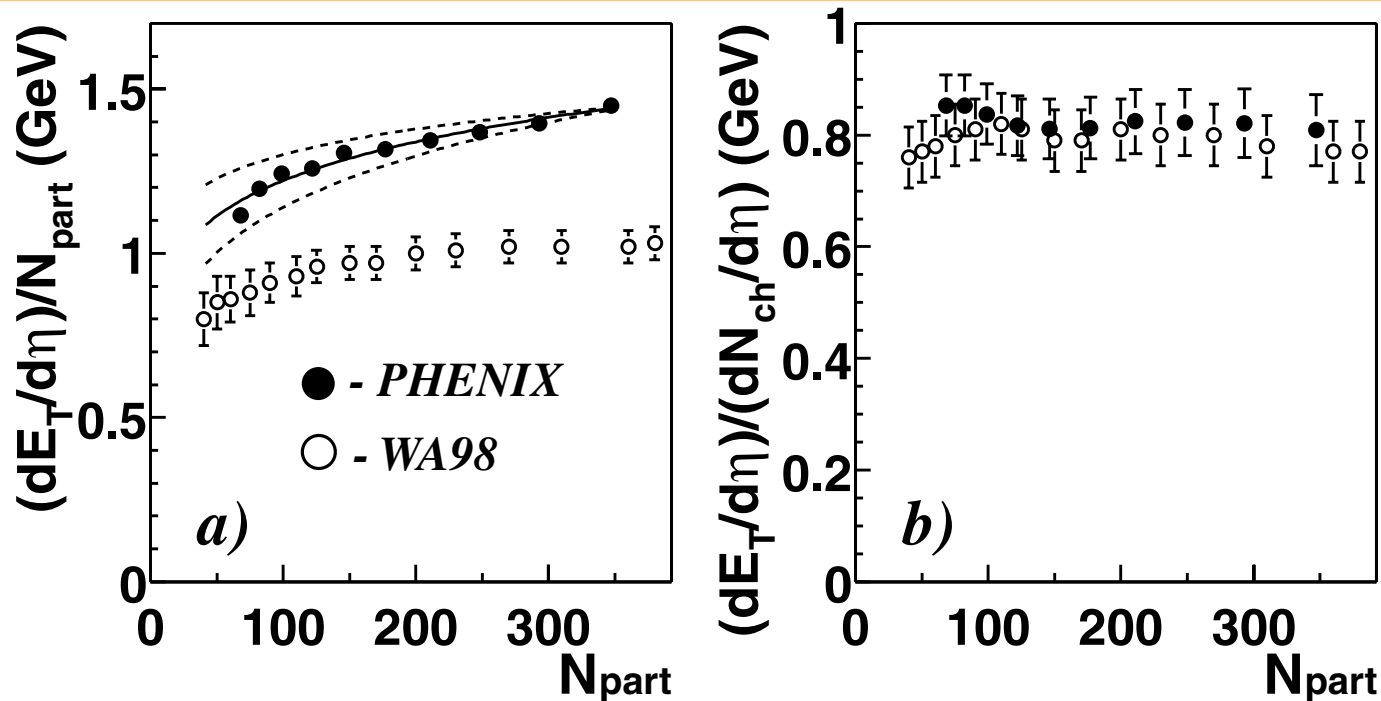
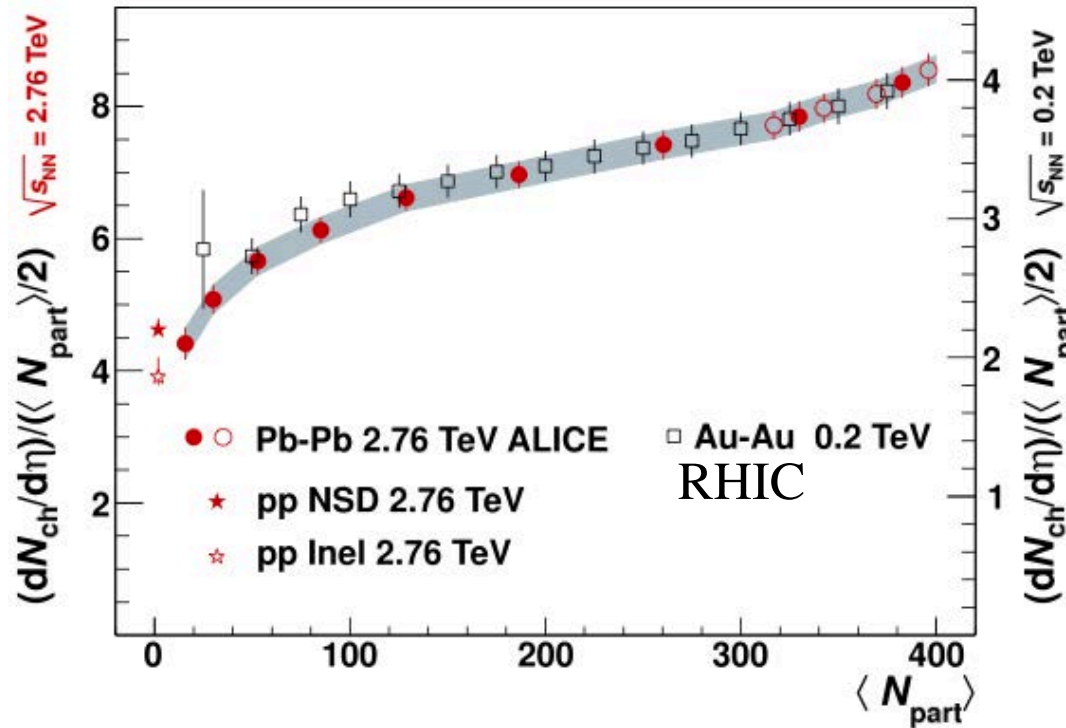


Fig. 4. (a) PHENIX transverse energy density per participant  $dE_T/d\eta/N_{part}$  for Au+Au at  $\sqrt{s_{NN}}=130$  GeV as a function of  $N_{part}$ , the number of participants, compared to the data of WA98 for Pb+Pb collisions at  $\sqrt{s_{NN}}=17.2$  GeV. The solid line is the  $N_{part}^\alpha$  best fit and the dashed lines represent the effect of the  $\pm 1\sigma$   $N_{part}$ -dependent systematic errors for  $dE_T/d\eta$  and  $N_{part}$ . **These are the Type B correlated systematic errors, all points move together by the same fraction of the systematic error at each point.**

# Important Observation 2.76 TeV cf. 200 GeV

ALICE  $\sqrt{s_{NN}}=2.76$  TeV  
PRL 106(2011)032301



- Exactly the same shape vs.  $N_{part}$  although  $\langle N_{coll} \rangle$  is a factor of 1.6 larger and the hard-scattering cross section is considerably larger.
  - ✓ PHENIX (2001)  $dN_{ch}/d\eta \sim N_{part}^\alpha$  with  $\alpha=1.16 \pm 0.04$  at  $\sqrt{s_{NN}}=130$  GeV
  - ✓ ALICE (2013)  $dN_{ch}/d\eta \sim N_{part}^\alpha$  with  $\alpha=1.19 \pm 0.02$  at  $\sqrt{s_{NN}}=2760$  GeV
- Strongly argues against a hard-scattering component and for a Nuclear Geometrical Effect.

# Previous analyses have shown that Quark Participant Model works in Au+Au but could have been the AQM

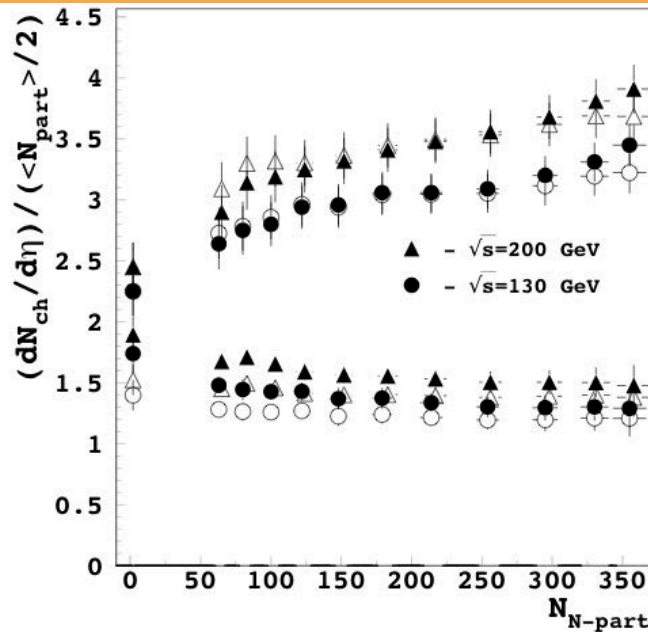
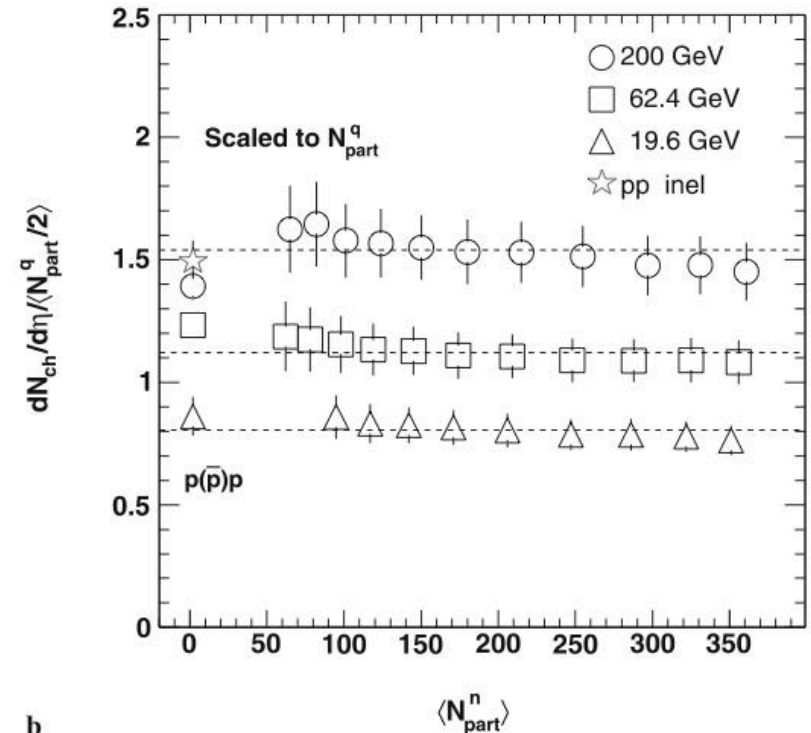


FIG. 3. (Color online)  $N_{ch}$  per nucleon and quark participant pair vs centrality. The results for quark participant pair are shown for  $\sigma_{qq} = 4.56$  mb (solid symbols) and  $\sigma_{qq} = 6$  mb (open symbols).

Eremin&Voloshin, PRC **67** (2003) 064905



b

Nouicer, EPJC **49** (2007) 281

These analyses didn't do entire distributions but only centrality-cut averages. Also they just generated 3 times the number of nucleons in a nucleus according to the Au nuclear density and called them constituent quarks then let them interact with the conventional q+q cross section  $\sigma_{q+q} = \sigma_{N+N}/9$ . The p+p result used constant radial density in a proton taken as a hard sphere with  $r < 0.8$  fm.

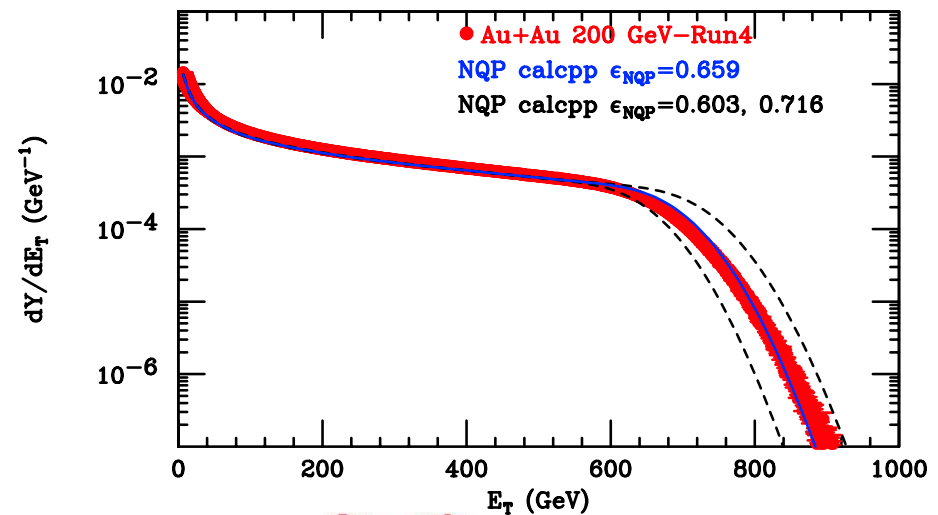
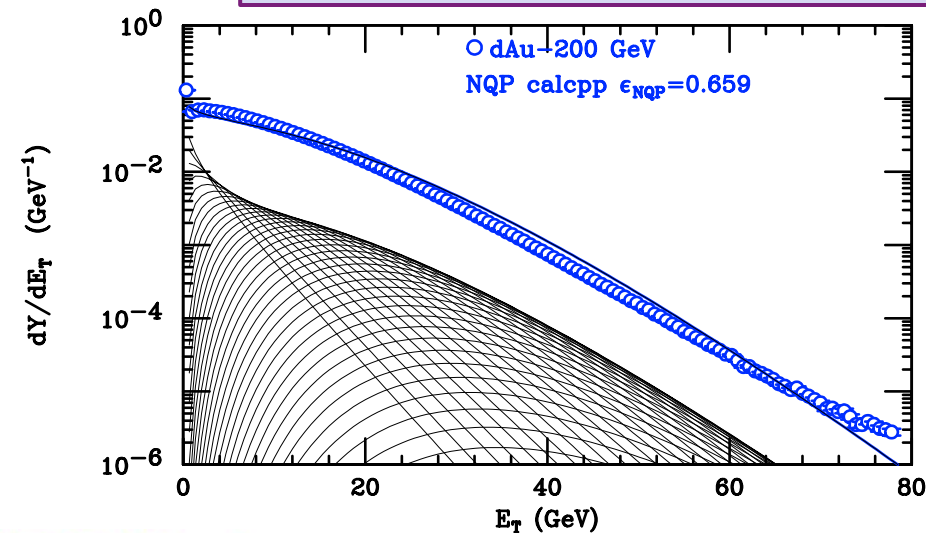
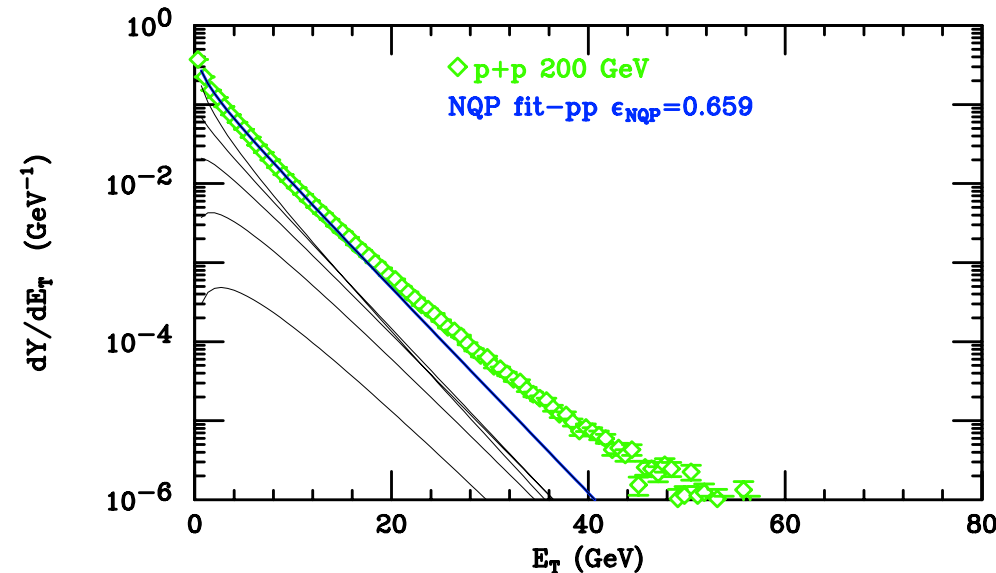
# PHENIX NQP model: Data driven $pp \rightarrow dAu, AuAu$

PHENIX2014  $E_T$  distributions  
from PRC89 (2014) 044905

1) Generate 3 constituent quarks around nucleon position, distributed according to proton charge distribution for pp, dA, AA

2) Deconvolute p-p  $E_T$  distribution to the sum of 2—6 quark participant (QP)  $E_T$  distributions taken as  $\Gamma$  distributions

3) Calculate dAu and AuAu  $E_T$  distributions as sum of QP  $E_T$  distributions



# Bill Zajc's explanation in PHENIX2014

Bill Zajc [now very senior] explained “the success of the two component model is not because there are some contributions proportional to  $N_{\text{part}}$  and some going as  $N_{\text{coll}}$ , but because a particular linear combination of  $N_{\text{part}}$  and  $N_{\text{coll}}$  turns out to be an empirical proxy for the number of constituent quarks”.

# How we generated the quarks around the nucleon position in PHENIX2014

PHENIX2014 [6], the spatial positions of the the three quarks were generated around the position of each nucleon in the Glauber monte carlo calculations for  $p + p$ ,  $d + \text{Au}$  and  $\text{Au} + \text{Au}$  collisions using the proton charge distribution corresponding to the Fourier transform of the form factor of the proton [24]:

$$d^3\mathcal{P}/d^3r = \rho^{\text{proton}}(r) = \rho_0^{\text{proton}} \times \exp(-ar), \quad (4)$$

where  $a = \sqrt{12}/r_m = 4.27 \text{ fm}^{-1}$  and  $r_m = 0.81 \text{ fm}$  is the r.m.s radius of the proton weighted according to charge [2]

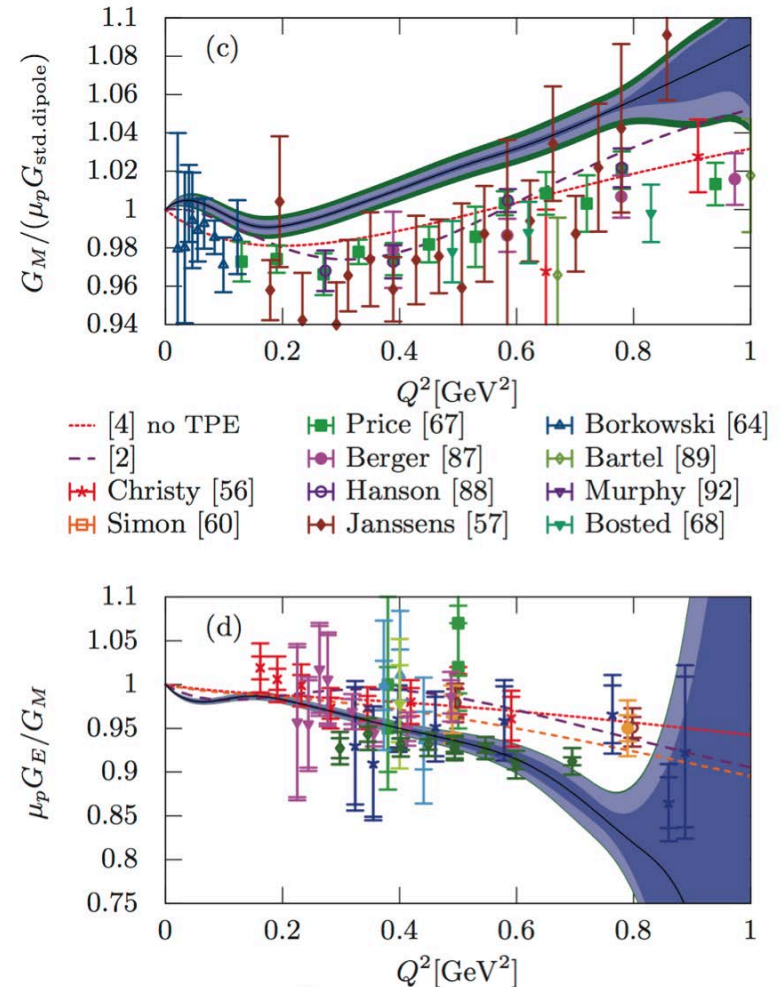
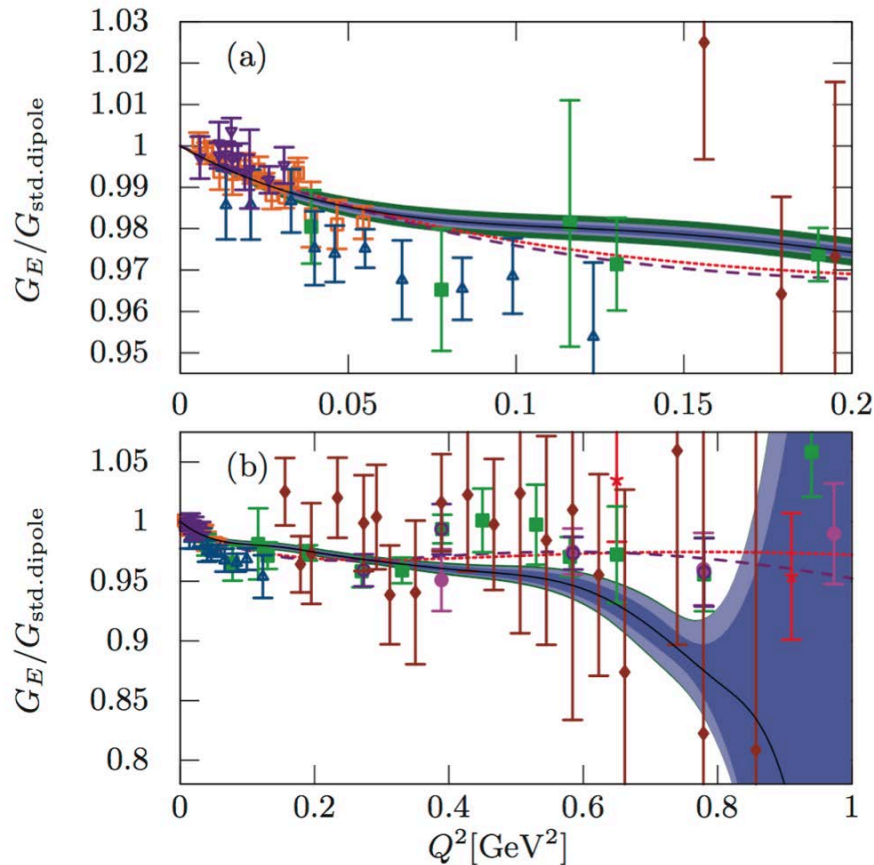
$$r_m = \int_0^\infty r^2 \times 4\pi r^2 \rho^{\text{proton}}(r) dr \quad . \quad (5)$$

The corresponding proton form factor is the Hofstadter dipole fit [25] now known as the standard dipole [26]:

$$G_E(Q^2) = G_M(Q^2)/\mu = \frac{1}{(1 + \frac{Q^2}{0.71\text{GeV}^2})^2} \quad (6)$$

Note that dipole fit agrees with  $G_E, G_M$  data to within a few % for  $Q^2 \leq 1 \text{ GeV}^2$ . The 'famous' radius anomaly is the upslope for  $Q^2 \leq 0.1$  in (b)

Mainz, Bernauer et al PHYSICAL REVIEW C **90**, 015206 (2014)



# I verified the dipole fit in my PhD thesis

## $\mu+p$ elastic scattering

errors. Tracks generated by the program were traced through the detection apparatus, and the simulated events were then put through the same reconstruction program as were the real events.

The characteristics of Monte-Carlo-generated events and real events were then compared in detail in order to establish the validity of the method. For example, it was verified that the simulated and real events gave the same distributions in the coplanarity and copunctuality variables. Most definitive, however, was

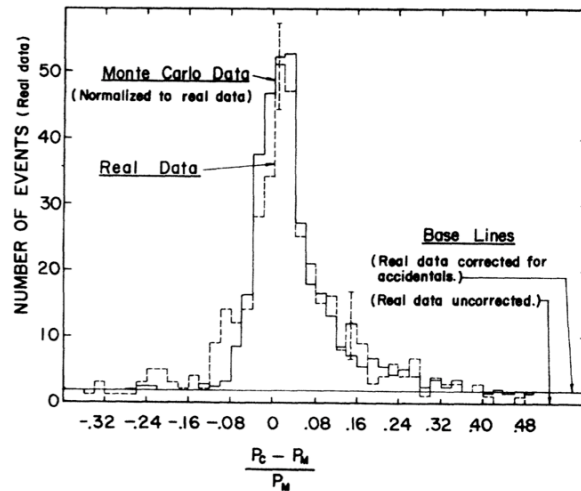


FIG. 2. Frequency distributions of real and simulated (Monte-Carlo) scattering events, versus  $(P_C - P_M)/P_M$ . The shift of baseline for the real events is the result of subtracting the accidental coincidences (see text), which have a flat distribution. The Monte-Carlo distribution has been normalized to have the same area as the real distribution after correction.

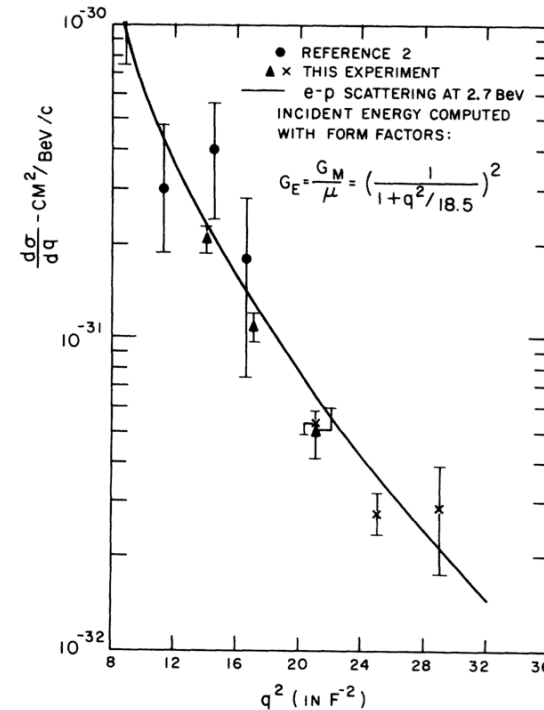


FIG. 3. Comparison of muon cross section ( $d\sigma/dq$ ) with the Rosenbluth prediction computed with a phenomenological fit to proton form factors. The two results at  $q^2 = 20 \text{ F}^{-2}$  represent the overlap of two separate runs, medium- $q^2$  and high- $q^2$ , differing in the thickness of absorber in front of the range chamber (Fig. 1). The expressions for  $G_E$  and  $G_M$  were chosen to fit low- $q^2$   $e-p$  scattering data [L. N. Hand, D. G. Miller, and Richard Wilson, Rev. Mod. Phys. **35**, 335 (1963)], and were found to fit data<sup>7</sup> with  $q^2 \geq 20 \text{ F}^{-2}$  as well as the more commonly used form made up of resonance terms. Results of earlier experiments are also shown.

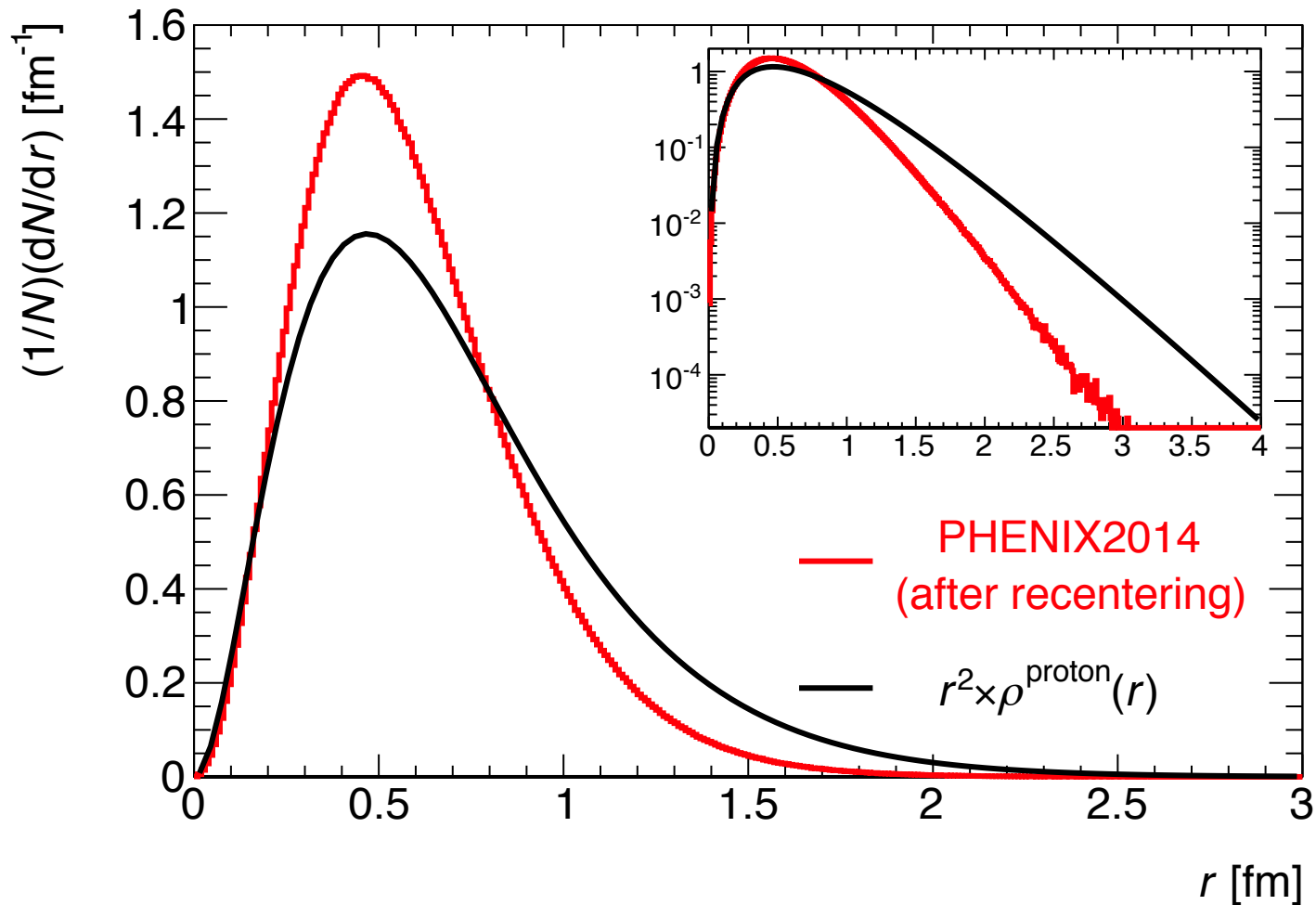
I even used  
a detector  
simulation

We got a comment from Adam Bzdak via Pete Steinberg 6 months after the paper appeared in PRC that our method didn't preserve the radial charge distribution about the c.m. of the three generated quarks

- This statement is correct so several of us got together to figure out how to generate 3 quarks about a nucleon that would preserve the c.m. position and the charge distribution about this c.m and how this would affect our results from PHENIX2014.

- We found 3 new methods that preserve both the original proton c.m. with the correct charge distributions about the c.m. “Planar Polygon”, “Explicit Joint”, “Empirical Recentered” I discuss 2. See Mitchell, Perepelitsa, Tannenbaum and Stankus PRC93,054910 (2016)

# Radial distribution of the quarks about the c.m. for PHENIX2014 compared to $r^2 \rho^{\text{proton}}(r) = r^2 \exp -4.27r$



# New centered Methods-PHENIX2014 data

PHYSICAL REVIEW C **93**, 054910 (2016)

## Tests of constituent-quark generation methods which maintain both the nucleon center of mass and the desired radial distribution in Monte Carlo Glauber models

J. T. Mitchell, D. V. Perepelitsa, and M. J. Tannenbaum  
*Brookhaven National Laboratory, Upton, New York 11973, USA*

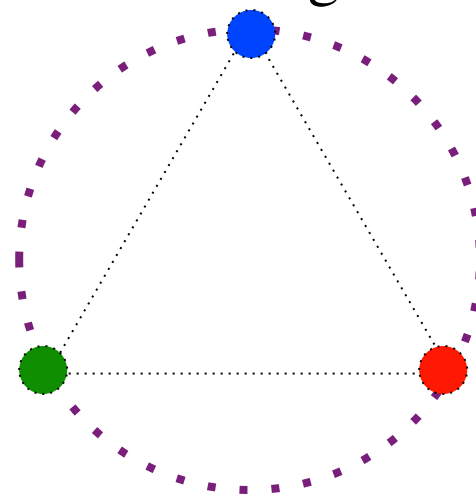
P. W. Stankus  
*Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*  
(Received 29 March 2016; published 23 May 2016)

Several methods of generating three constituent quarks in a nucleon are evaluated which explicitly maintain the nucleon's center of mass and desired radial distribution and can be used within Monte Carlo Glauber frameworks. The geometric models provided by each method are used to generate distributions over the number of constituent quark participants ( $N_{qp}$ ) in  $p + p$ ,  $d + Au$ , and  $Au + Au$  collisions. The results are compared with each other and to a previous result of  $N_{qp}$  calculations, without this explicit constraint, used in measurements of  $\sqrt{s_{NN}} = 200$  GeV  $p + p$ ,  $d + Au$ , and  $Au + Au$  collisions at the BNL Relativistic Heavy Ion Collider.

DOI: [10.1103/PhysRevC.93.054910](https://doi.org/10.1103/PhysRevC.93.054910)

# Planar Polygon

Generate one quark at  $(r,0,0)$  with  $r$  drawn from  $r^2 e^{\{-4.27r\}}$ . Then instead of generating  $\cos \theta$  and  $\Phi$  at random and repeating for the two other quarks as was done by PHENIX2014, imagine that this quark lies on a ring of radius  $r$  from the origin and place the two other quarks on the ring at angles spaced by  $2\pi/3$  radians. Then randomize the orientation of the 3-quark ring spherically symmetric about the origin. This guarantees that the radial density distribution is correct about the origin and the center of mass of the three quarks is at the origin but leaves three quark triplet on each trial forming an equilateral triangle on the plane of the ring.



# DVP—Empirical Radial distribution Recentered

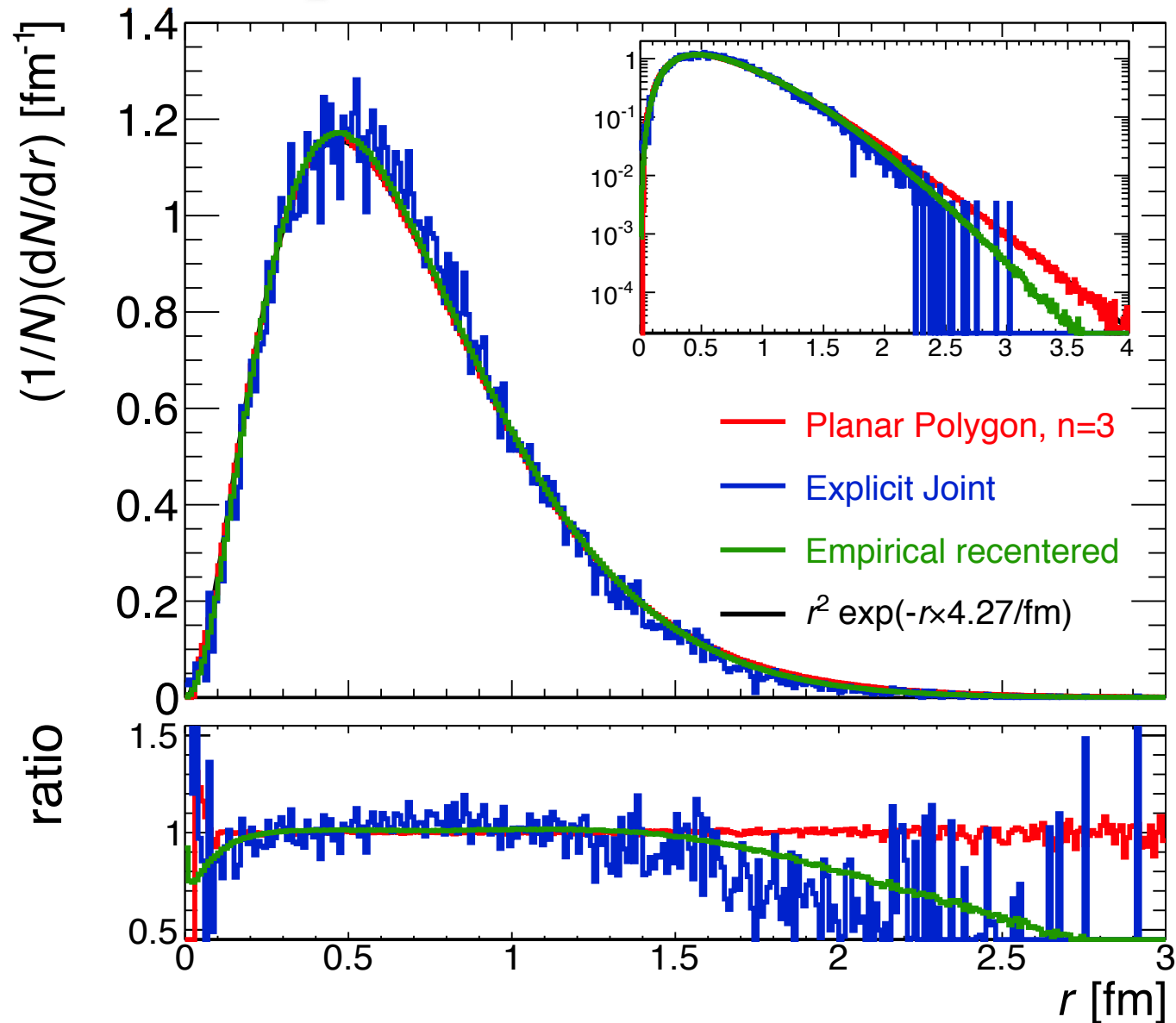
$$f(r) = r^2 \rho(r) = r^2 e^{-4.27r} (1.21466 - 1.888r + 2.03r^2) \\ (1 + 1.0/r - 0.03/r^2)(1 + 0.15r)$$

where  $r$  is the radial position of the quark in fm.

- the three constituent-quark positions are drawn independently from the auxiliary function  $f(r)$  above. Then the center of mass of the generated three-quark system is re-centered to the original nucleon position.

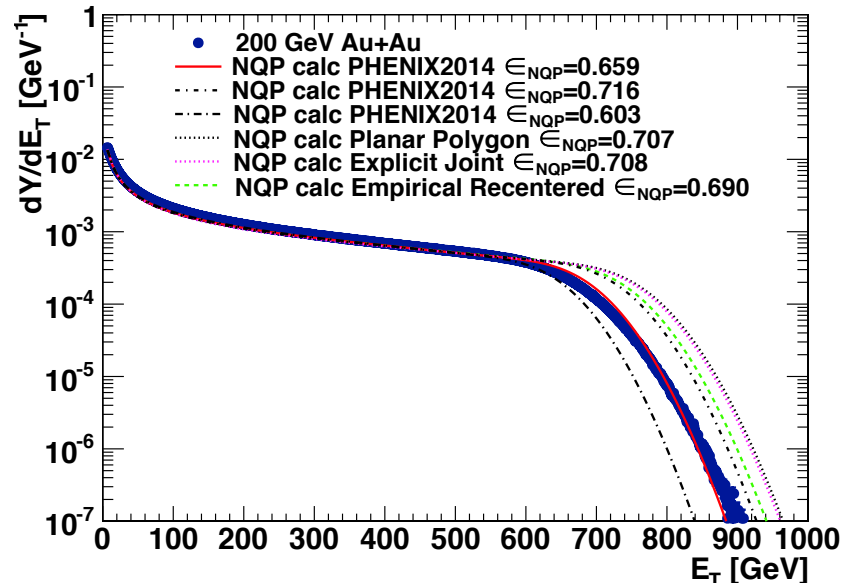
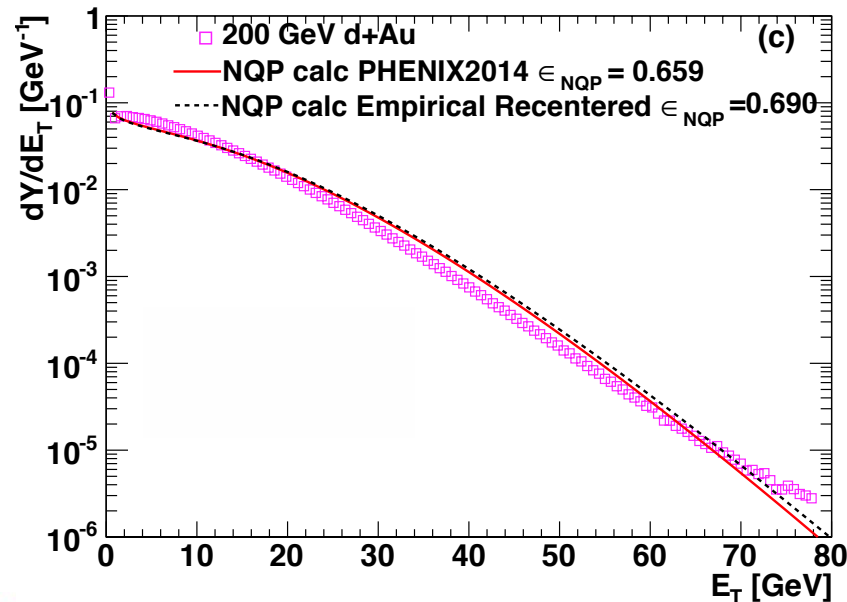
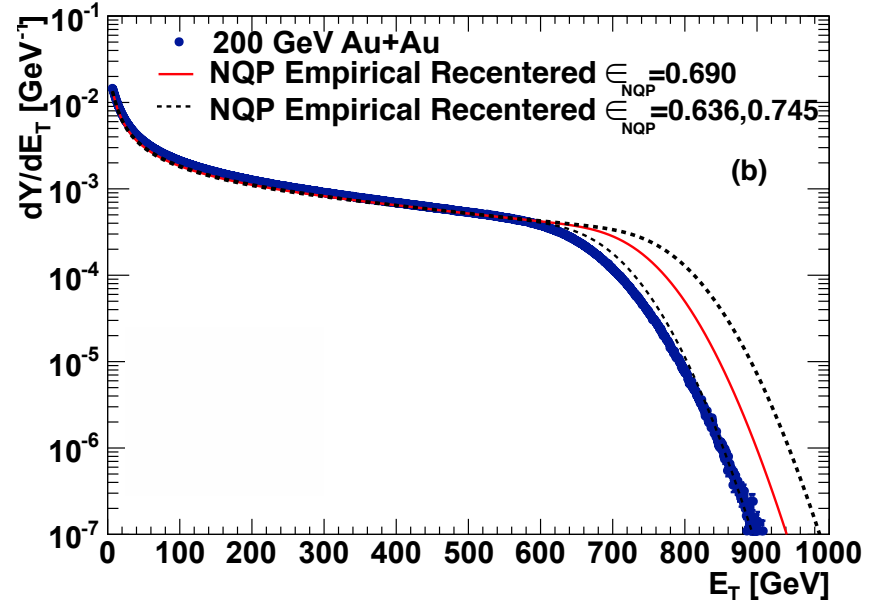
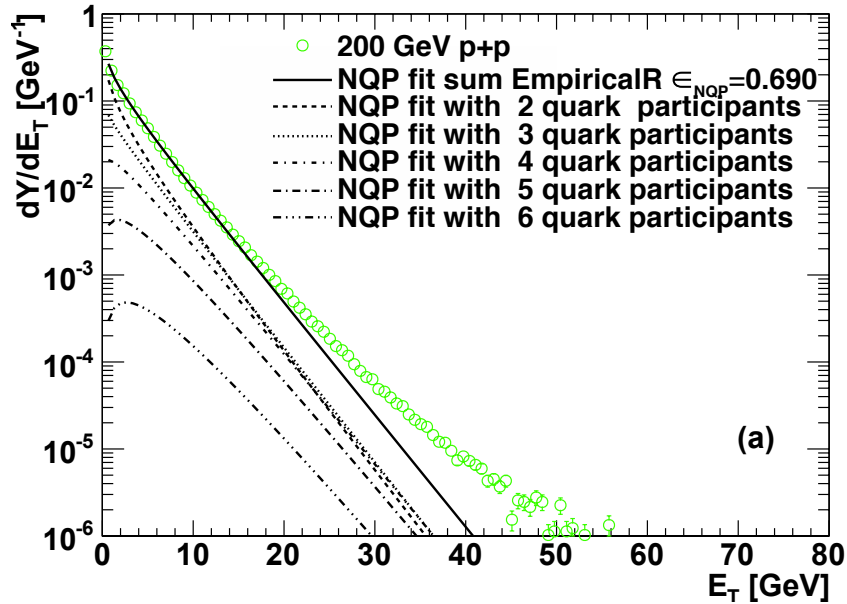
This function was derived through an iterative, empirical approach. For a given test function  $f^{\text{test}}(r)$ , the resulting radial distribution  $q^{\text{test}}(r)$  was compared to the desired distribution  $q^{\text{proton}}(r)$  in Eq. 4. The ratio  $q^{\text{test}}(r) / q^{\text{proton}}(r)$  was parameterized with a polynomial function of  $r$  or  $1/r$ , and the test function was updated by multiplying it with this parametrized functional form. Then, the procedure was repeated with the updated test function used to generate an updated  $q^{\text{test}}(r)$  until the ratio  $q^{\text{test}}(r) / q^{\text{proton}}(r)$  was sufficiently close to unity over a wide range of  $r$  values.

# New Radial quark distributions about the c.m.

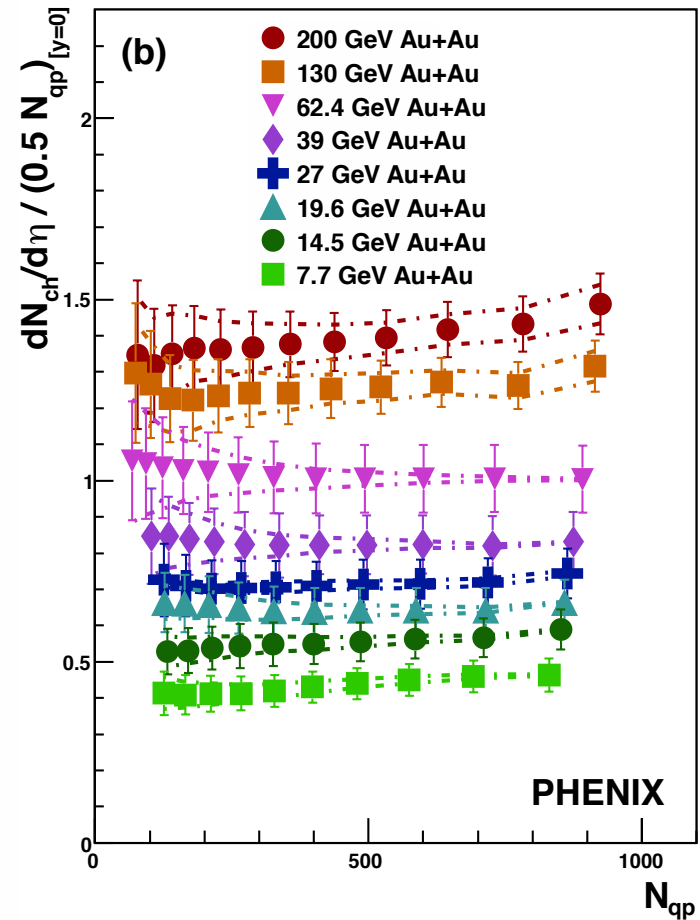
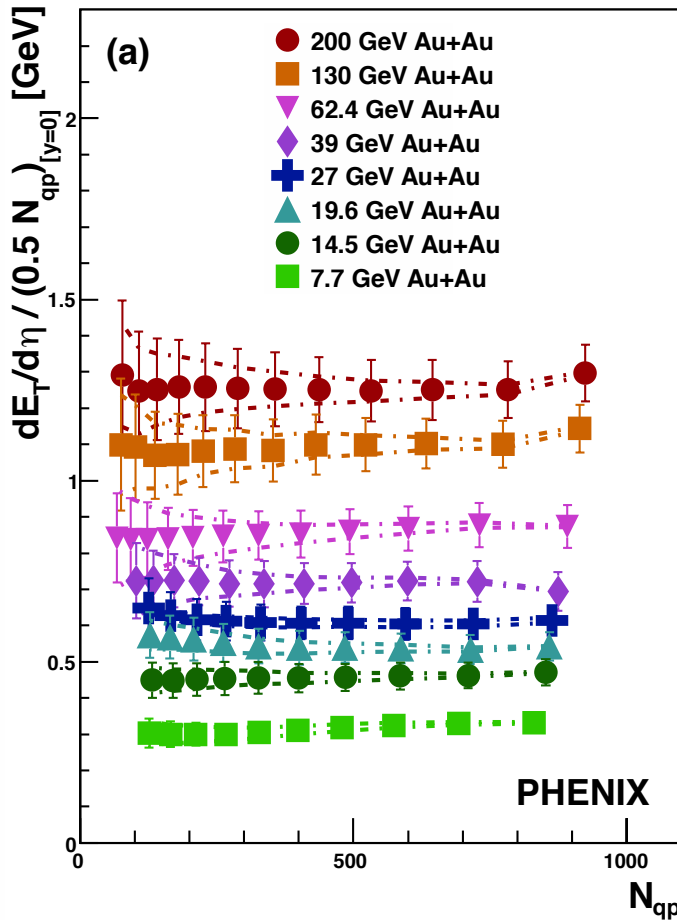


# NQP centered with PHENIX2014 data

all work within  $\sim 1$  standard deviation



# Constituent-quark-participant scaling vs. $N_{qp}$

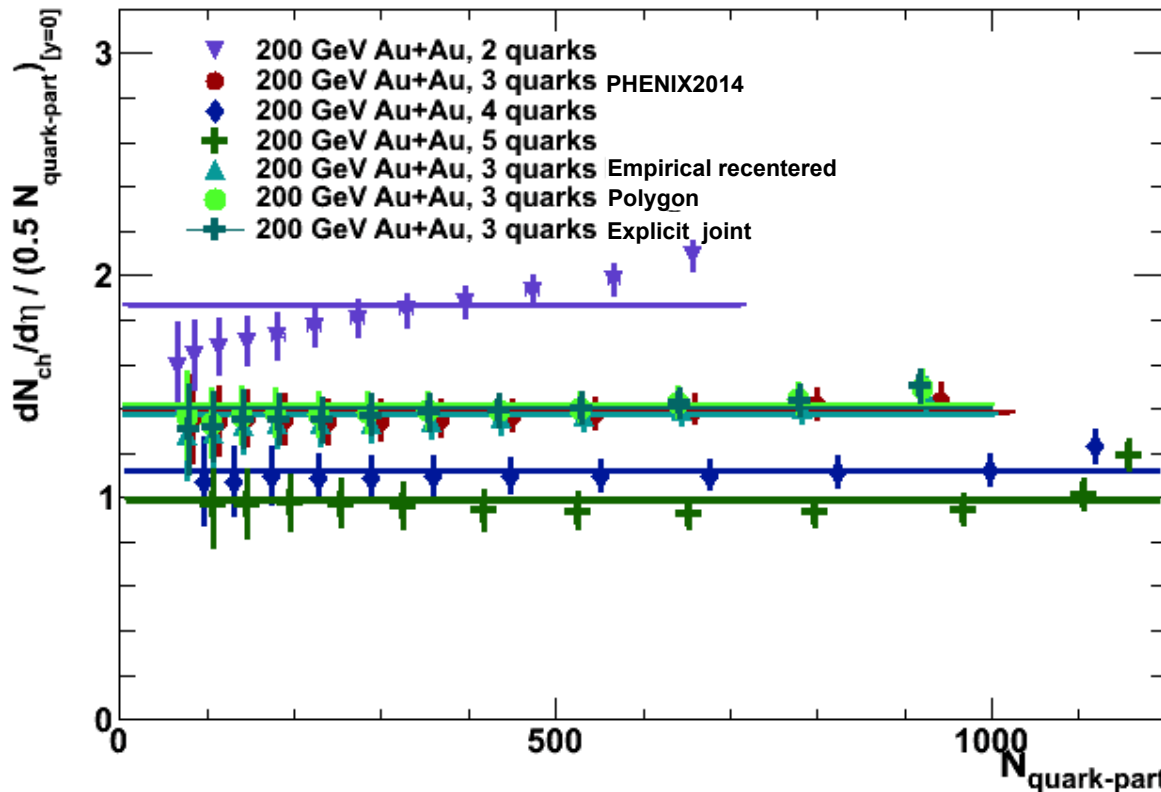


PHENIX PRC93(2016)024901

Uses Empirical Recentered---now standard

# What happens with 2,3,4,5 quarks

- since many people had asked why do you stop at 3 quarks: why not 2, 4, 5, we looked into this too with  $(dN_{ch}/d\eta)/0.5N_{qp}$



- 2 is rejected; 3 give the same result for all 4 methods; 4,5 seem to work as well as 3 in the PHENIX2014 calculation

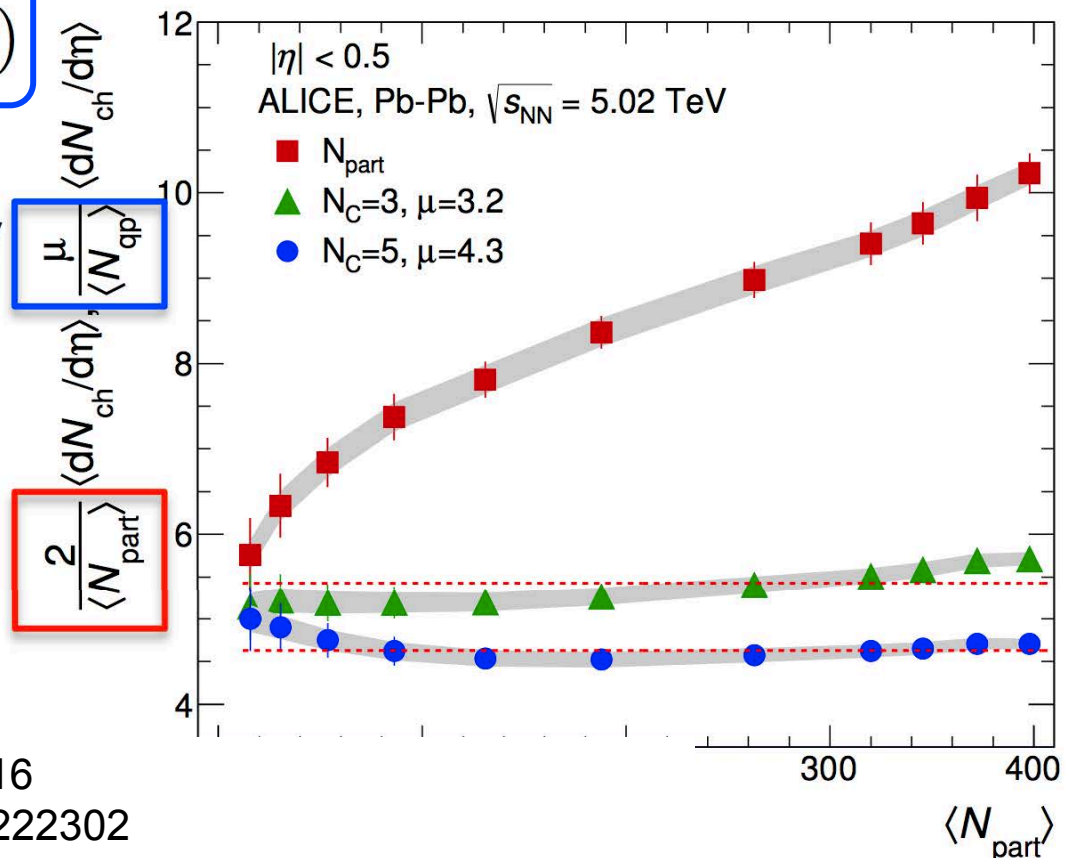
# Agreement from ALICE

## Glauber MC with quark scaling [10]

Single quark position determined with proton density:

$$\rho(r) = \rho_0^{proton} \exp(-a \cdot r)$$

particle multiplicity [9]  
density **scales linearly**  
with the number of  
constituent quark  
participants [8]

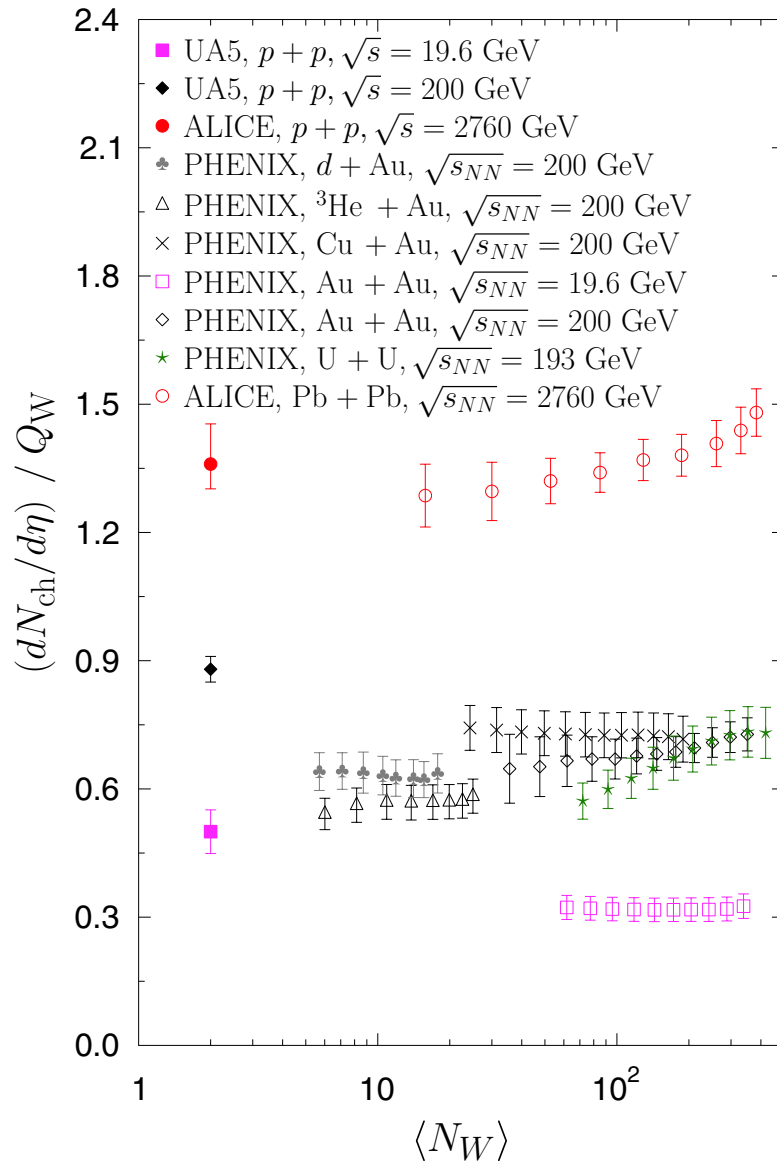


[8] **ALICE** Collaboration, V. Zaccolo, IS2016

[9] **ALICE** Collaboration, PRL **116** (2016) 222302

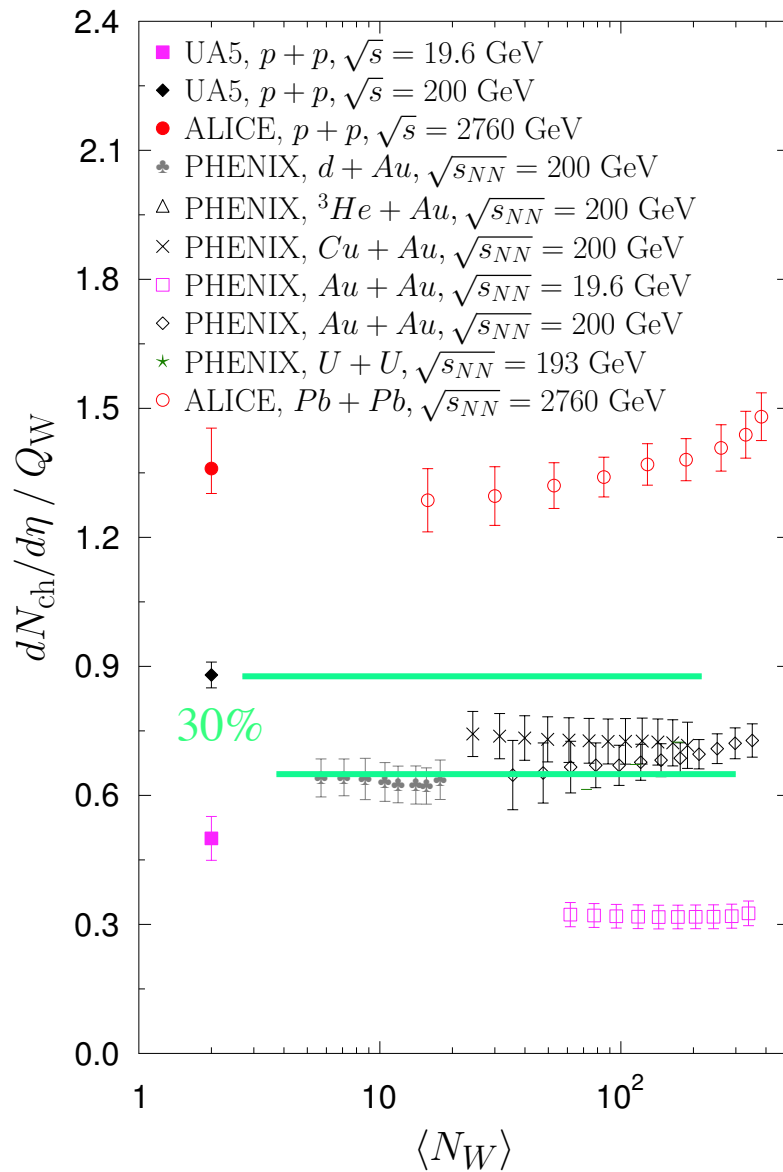
[10] C. Loizides, PRC **94** (2016) 024914-Uses Empirical Recentered Formula

# Disagreement from another NQP calculation?



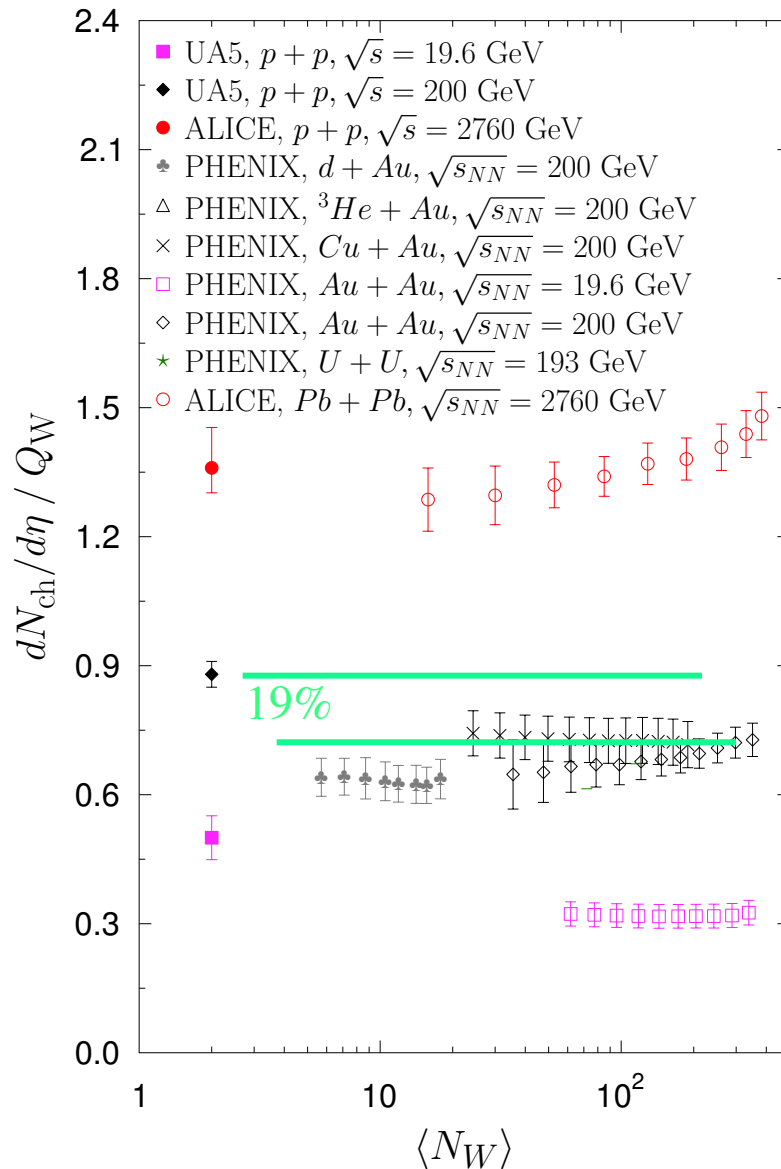
Bozek, Broniowski, Rybczynski  
PRC94(2016)014902 do a constituent quark participant calculation which they call  $Q_W$  (wounded quark) and find that it works for ALICE Pb+Pb  $\sqrt{s_{NN}}=2.76$  TeV but “At lower collision energies, such as  $\sqrt{s_{NN}} = 200$  GeV, the universality is far from perfect and the obtained scaling is approximate, exhibiting some dependence on the reaction. Moreover, we note in Fig. 1 that the corresponding  $p + p$  point is higher by about 30% from the band of other reactions.”

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# Disagreement from another NQP calculation?



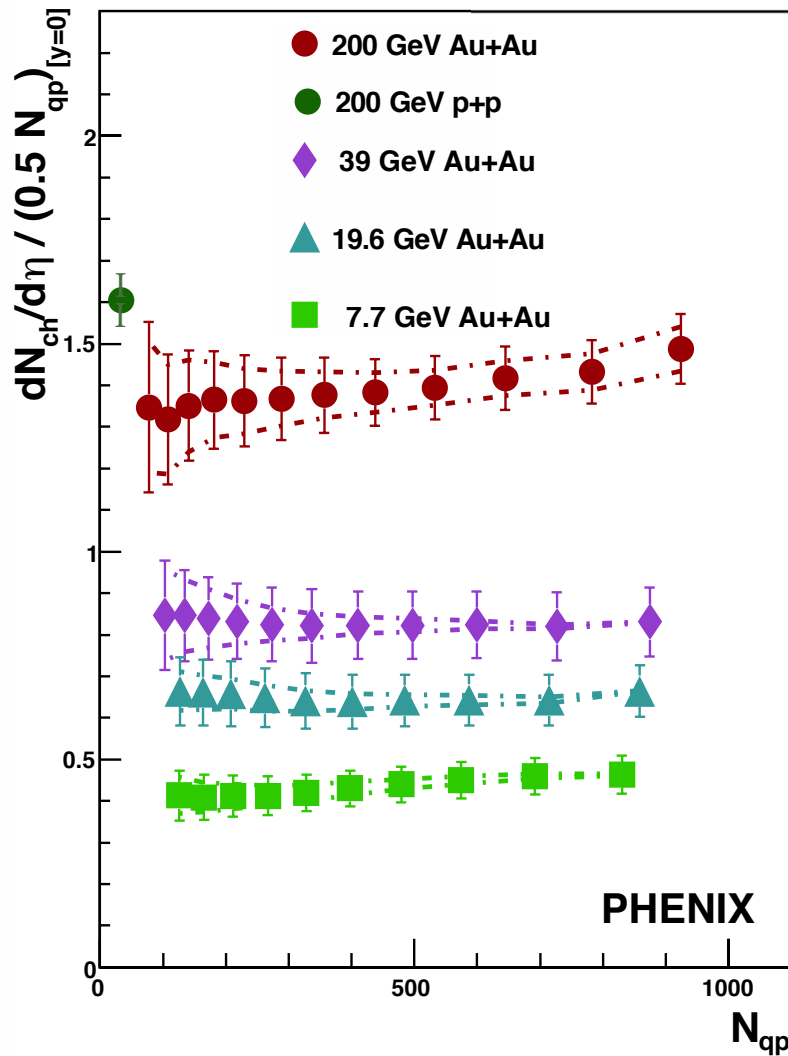
“Moreover, we note in Fig. 1 that the corresponding  $p + p$  point is higher by about 30% from the band of other reactions.” (only from one AuAu point)

Of course I noted that they only used our tabulated statistical errors but left out our Type B correlated systematics shown on our plots where **all** the data points can be moved up to the top of their syserror bars with the cost of  $1 \sigma$ , so that the ratio of the  $p+p$  to lowest AuAu point is  $1.19 \pm 0.17$  statistical, or  $1.33 \pm 0.22$  if we simply add the sys and stat in quadrature. i.e.  $33 \pm 22\% \approx 30\%$

But this difference is not significant.

# Disagreement from another NQP calculation?

## Here is our calculation.



We actually didn't calculate the p+p value in PRC93 (2016) 024901, but did show the systematic errors on the plot. So here they are along with the p+p calculation from PRC93 (2016) 054910 using the same UA5 pbar+p  $dN_{ch}/d\eta = 2.23 \pm 0.08$  at  $\sqrt{s}=200$  GeV with a p+p/Au+Au ratio of  $1.19 \pm 0.19 \pm 0.16$  sys i.e. agreement to  $\approx 1 \sigma$  for all the data points at 200 GeV Au+Au.

As far as I can tell BB&R use  $r_m=0.94$  fm for the proton rms radius in Eq 4 and a gaussian wounding profile for a q+q collision--Not the standard Glauber.

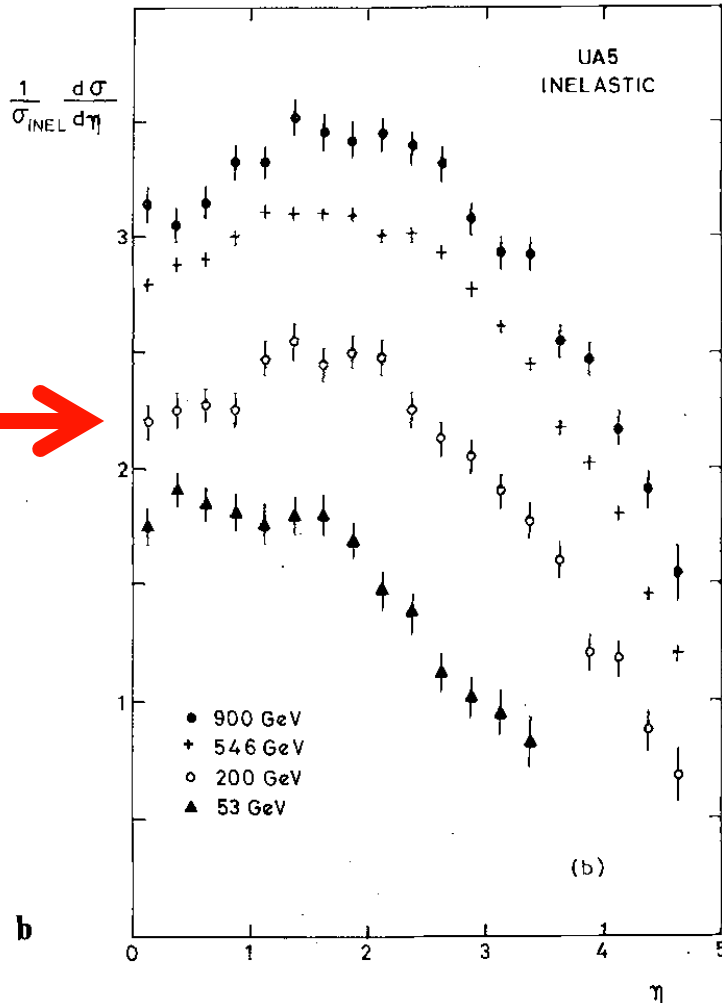
# Conclusions

- The Constituent Quark Participant Model ( $N_{qp}$ ) works at mid-rapidity for A+B collisions in the range ( $\sim 20$  GeV)  $39 \text{ GeV} < \sqrt{s_{NN}} < 5.02 \text{ TeV}$ .
- Experiments generally all use the same Glauber M.C. but the BB&R's M.C. is different for q+q scattering leading to somewhat different results.
- Attention must be paid to systematic errors.
- How can the event-by-event proton radius variations and quark-quark correlations used in Constituent Quark Glauber models be measured?

# EXTRAS

# UA5 $dN_{ch}/d\eta$ inelastic pbar+p

$\sqrt{s}=200$  GeV  $\rightarrow$



**Fig. 1 a, b.** Pseudorapidity distributions obtained by UA5 at various energies from the ISR and the CERN Collider for **a** non single-diffractive (NSD) events, **b** inelastic (i.e. NSD + SD) events

# Details on “Disagreement” of NQP calculations

Table 1:  $N_{qp}$  in p+p


	paper	$\sqrt{s_{NN}}$	$\sigma_{nn}^{inel}$	$r_m$	$\sigma_{qq}^{inel}$ (mb)	$\langle N_{qp} \rangle$
	p+p	(GeV)	(mb)	(fm)	(GeV)	
PX2014 Phys. Rev. C	<b>89</b> , 044905 (2014)	200	42.0	0.81	9.36	2.99
MPTS Phys. Rev. C	<b>93</b> , 054910 (2016)	200	42.3	0.81	8.17	2.78
Loizides Phys. Rev. C	<b>94</b> , 024914 (2016)	200	42.	0.81	8.1	2.8
BB&R Phys. Rev. C	<b>94</b> , 014902 (2016)	200	41.3	0.94	7.0	2.60

reaction	dn/deta	err	sys	QW	err	
p+p Bozek	2.29	0.08		2.6		
p+p MJT Bozek	2.23	0.08		2.6		
p+p MPTS	2.23	0.08		2.78		
cent 55-60				QW	err	
AuAu Bozek	52.2	6.5	4.88	80.65		
AuAu PX	52.2	6.5	4.88	77.5	6.8	
	dnch/QW	err				
p+p Bozek	0.881	0.031				
p+p MJT Bozek	0.858	0.031				
p+p MPTS	0.802	0.029				
	dnch/QW	stat	sys			
AuAu Bozek	0.647	0.081	0.061			
AuAu PX	0.674	0.103	0.086			
		stat	sys	stat+sys	shift sys	stat
pp/Au Bozek	<b>1.361</b>	0.176	0.136	<b>0.222</b>	<b>1.225</b>	<b>0.176</b>
ppmjtB/AuB	<b>1.325</b>	0.172	0.133	<b>0.217</b>	<b>1.192</b>	<b>0.172</b>
pp/AuAu PX	<b>1.191</b>	0.186	0.159	<b>0.245</b>	<b>1.032</b>	<b>0.186</b>

# Details of NQP calculation PHENIX2014

## The NQP calculation for a B+A reaction

$$\left(\frac{d\sigma}{dE_T}\right)_{\text{NQP}} = \sigma_{BA} \sum_{n=1}^B w_n P_n(E_T) \quad (15)$$

- $\sigma_{BA}$  is the measured B+A cross section in the detector aperture,
- $w_n$  is the relative probability for  $n$  quark participants in the B+A reaction from a Glauber Monte Carlo. 
- $P_n(E_T)$  is the calculated  $E_T$  distribution on the detector aperture for  $n$  **independently interacting** quark participants.
- If  $f_1(E_T)$  is the measured  $E_T$  spectrum on the detector aperture for one quark participant, and  $p_0$  is the probability for the elementary collision to produce no signal on the detector aperture, then, the correctly normalized  $E_T$  distribution for one quark participant is:

$$P_1(E_T) = (1 - p_0)f_1(E_T) + p_0\delta(E_T) \quad , \quad (16)$$

where  $\delta(E_T)$  is the Dirac delta function and  $\int f_1(E_T) dE_T = 1$ .

- $P_n(E_T)$  (including the  $p_0$  effect) is obtained by **convoluting**  $P_1(E_T)$  with itself  $n - 1$  times

$$P_n(E_T) = \sum_{i=0}^n \frac{n!}{(n-i)! i!} p_0^{n-i} (1 - p_0)^i f_i(E_T) \quad (17)$$

where  $f_0(E_T) \equiv \delta(E_T)$  and  $f_i(E_T)$  is the  $i$ -th convolution of  $f_1(E_T)$ :

$$f_i(x) = \int_0^x dy f(y) f_{i-1}(x - y) \quad . \quad (18)$$

Apart from generating the positions of the 3 quarks per nucleon this is standard method for calculations of  $E_T$  distributions. See PHENIX PRC89(2014)044905 for further details. Also see MJT PRC69(2004)064902

3 quarks are distributed about the center of each nucleon with a spatial distribution  $\rho(r) = \rho(0) \exp(-ar)$  where  $a = \sqrt{12}/r_m = 4.27 \text{ fm}^{-1}$  and  $r_m = 0.81 \text{ fm}$  is the rms charge radius of the proton. [Hofstadter RevModPhys 28\(1956\)214](#)  
**Or as in the three new centered methods.**  
 The q-q inelastic scattering cross section is adjusted to 9.36 mb to reproduce the 42 mb N+N inelastic cross section at  $\sqrt{s_{NN}} = 200 \text{ GeV}$

Gamma distribution is used because it fits and because n-th convolution is analytical

$$f(x) = \frac{b}{\Gamma(p)} (bx)^{p-1} e^{-bx}$$

$$f_n(x) = \frac{b}{\Gamma(np)} (bx)^{np-1} e^{-bx}$$

# PWS, PS Explicit Joint distribution

$$P(\vec{x}_1, \dots, \vec{x}_n) = f(\vec{x}_1) f(\vec{x}_2) \dots f(\vec{x}_n) \delta \left( \sum \vec{x}_i / n \right)$$

- $\vec{x}_i$  are the vectors of the generated quarks from the origin and the delta function ensures that the c.m. of the vector sum stays at 0

(I) Let  $f^{[k]}(\vec{x})$  be defined as the  $k^{\text{th}}$ -order 3-D convolution of  $f(\vec{x})$  with itself; e.g.  $f^{[2]}(\vec{x}) = f(\vec{x}) \circ \circ \circ f(\vec{x})$ . Then it can be shown from Eq. 7 that for the singles distribution of any individual  $\vec{x}_i$  to follow  $\rho(\vec{x})$  the auxiliary function  $f()$  must satisfy

$$\rho(\vec{x}) = f(\vec{x}) f^{[n-1]}(-\vec{x}) \quad (8)$$

- even for the simple  $\rho(r) = e^{-4.27r}$  it is not straightforward to solve Eq.8 so they go to trial and error to see which  $f$  gives the best  $\rho$

# PWS, PS Explicit Joint distribution cont'd

- They find the  $f(x)$  which they claim matches the correct  $\rho(r)$  to within a few percent out to  $r < 2.3 \text{ fm}$

$$f(\vec{x}) = \exp - (b r / r_0) \left[ 1 + \left( \frac{r}{c r_0} \right) \right]$$

- with  $c=3.9$  and  $b=0.91$  and  $r_0 = 1/a = r_m / \sqrt{12} = 0.234 \text{ fm}$ .

Here's where the PWS and PS methods differ although both use rejection sampling

## Pete's rejection sampling

- (1) Select each  $\vec{x}_i$  independently according to the auxiliary distribution function  $f(\vec{x})$ , then
- (2) Calculate the center of mass coordinate  $x_{CM} = \Sigma \vec{x}_i / n$  for those values, and keep the sample if and only if  $x_{CM}$  is within some tolerance limit of zero, to enforce the effect of the delta function  $\delta(\Sigma \vec{x}_i / n)$ .

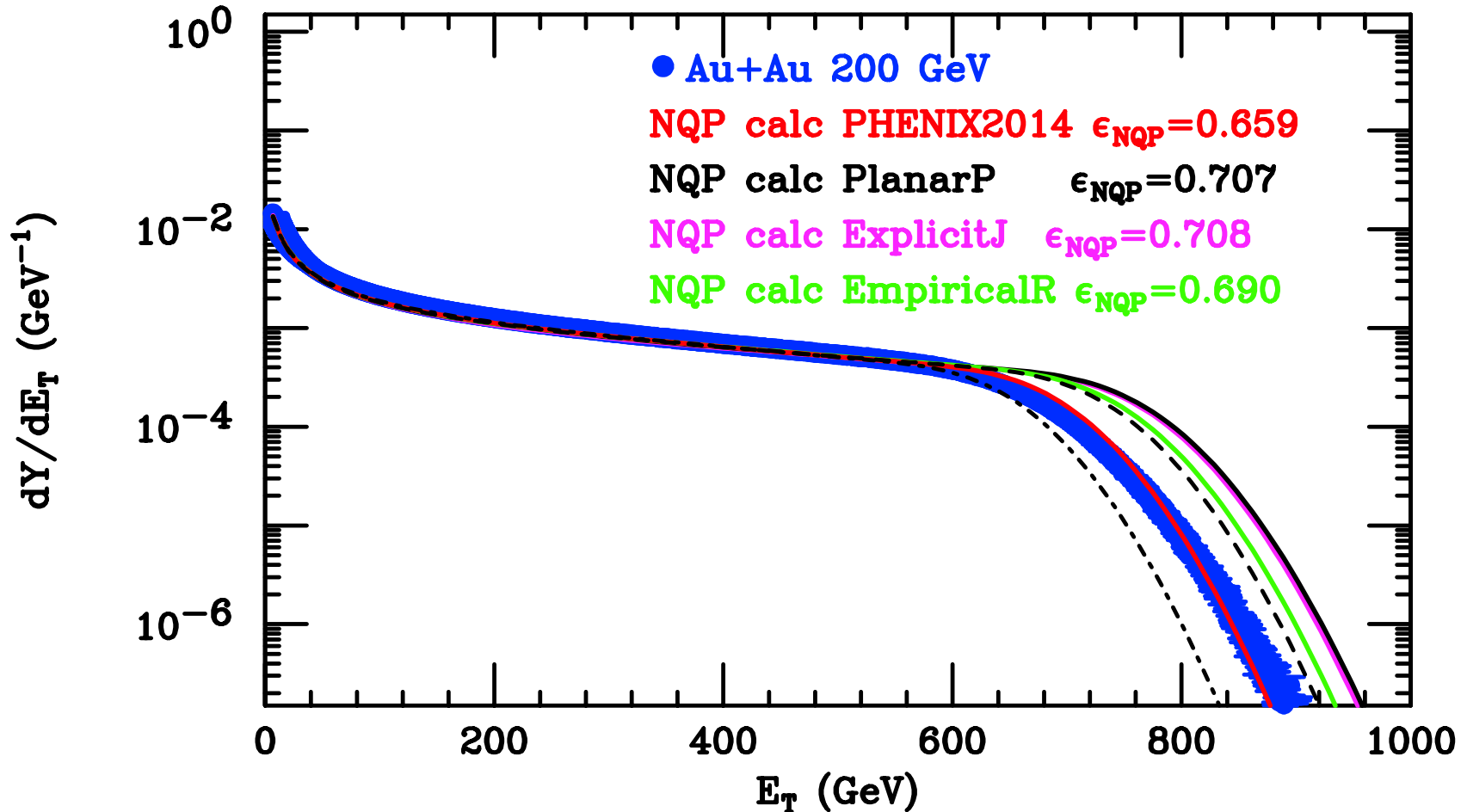
This is inefficient, generate lots of events to get a few good ones

## Paul's rejection sampling

- (1) Generate all but one of the vectors  $\vec{x}_i$  independently, each according to  $f(\vec{x})$ ; which one of the list is not chosen here is unimportant, let's suppose it is  $\vec{x}_1$ ; then
- (2) Calculate the value of the remaining vector as the negative sum of all the previously chosen ones, *e.g.*  $\vec{x}_1 = -(\vec{x}_2 + \dots + \vec{x}_n)$  to enforce the center of mass at zero; then
- (3) Keep this sample if and only if a new random number chosen uniformly on  $[0, 1]$  is less than or equal to the probability density of the final vector,  $f(\vec{x}_1)$ ; otherwise reject the sample and try again.

This keeps 10-20% of the events without degrading with increasing  $n$

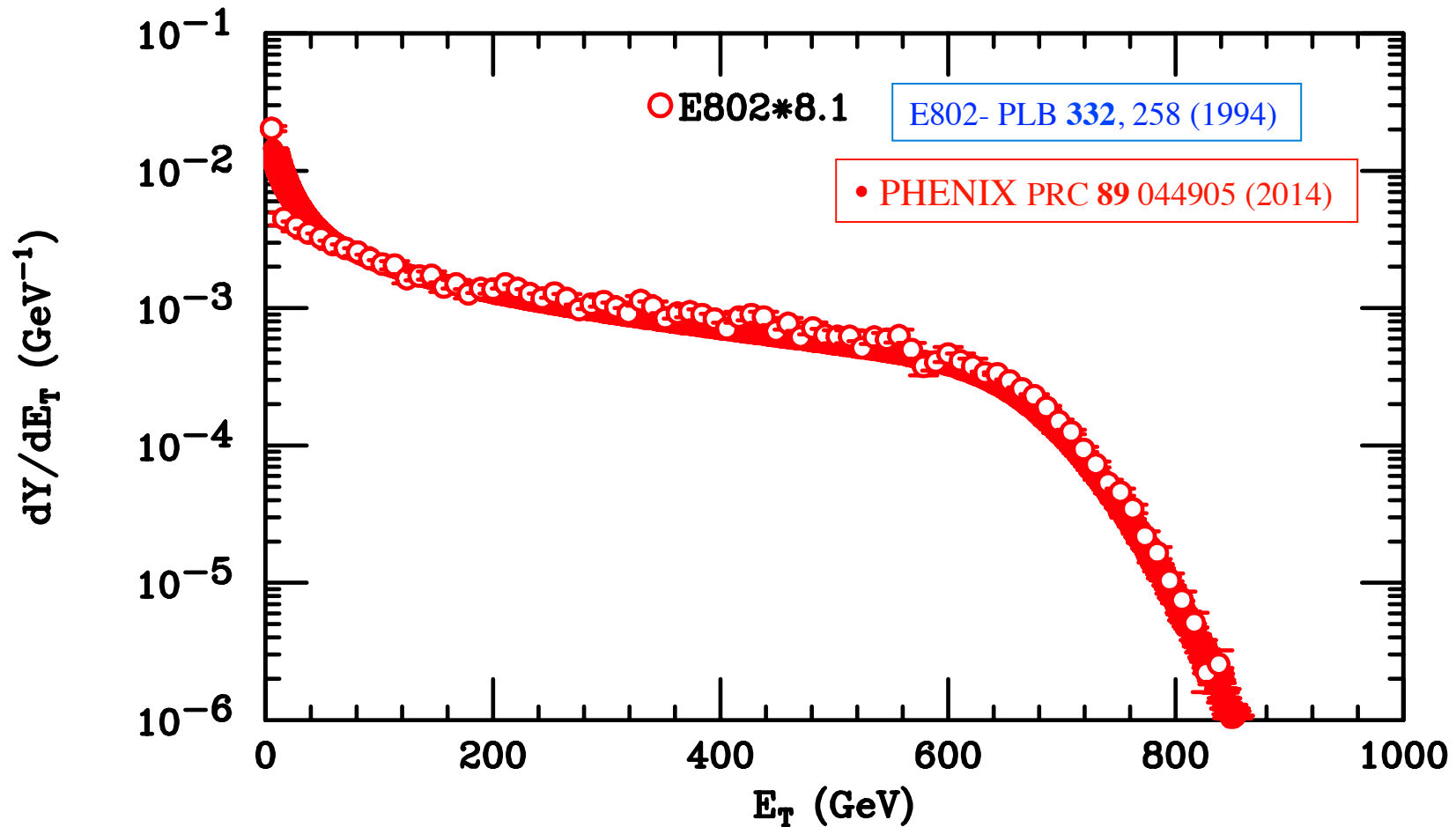
# Au+Au calculation all methods



surprisingly the most complicated Explicit J and simplest Planar P are virtually identical. EmpiricalR is  $\sim$ within  $1\sigma$  of PX2014

# Au+Au $E_T$ spectra at AGS $\sqrt{s_{NN}}=5.4$ GeV and RHIC 200 GeV are the same shape!!!

PHENIX and E802  $E_T$  Transverse Energy corr to  $\Delta\eta=1$   $\Delta\phi=2\pi$



But following the style of the CERN fixed target results at c. 2000, we stopped plotting distributions [PRL87,052301(2001)] and gave results as  $(dE_T/d\eta)/(0.5N_{part})$  vs.  $N_{part}$

